

## Robust optimization of a basketball free throw

To demonstrate the capabilities of Robustimizer, the basketball free throw example is used. The design variables for this problem are the throw speed ( $V$ ) and angle of throw ( $\theta$ ) and we assign those inputs using upper and lower bounds mentioned in Table 1.

Table 1. Lower and upper bound for design variables

Variable name	Lower bound	Upper bound
$V$ (m/s)	6	8
$\theta$	60	70

The noise parameters are magnitude of front wind ( $W$ ) and the throw height ( $y$ ) as shown in Figure 1 and they are described using their mean and standard deviation as expressed in Table 2.

Table 2. Mean and standard deviation for noise variables

Variable name	Mean	Standard deviation
$W$ (m/s)	3	1
$Y$ (m)	2.3	0.1

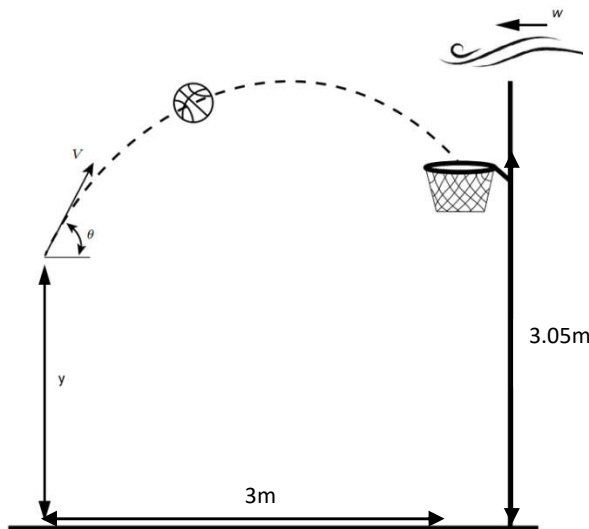


Figure 1. Schematic of the side view of a free throw

To define an output in this problem that determines the success of throw, we use the following criterion<sup>1</sup>:

$$p = \frac{(x_f - 3)^2}{d_x^2} \quad (1)$$

In which:

$$d_x = D - \frac{2r}{\sin(\beta)} \quad (2)$$

$D$ ,  $r$ ,  $d_x$ , and  $\beta$  are shown in Figure 2. A higher throw results in a higher angle of approach and therefore a wider range of success. On the other hand, a higher throw increases the effect of the wind and therefore results in more scatter of the final coordinates. It is expected that a strategy can be found that maximizes the number of successful throws. The lower the value for  $p$  the more the success of the throw. The goal is to obtain a setting of design parameters in which  $p$  has minimum value and variation in the windy situation and for a variety of throw heights which guarantees the highest rate of success.

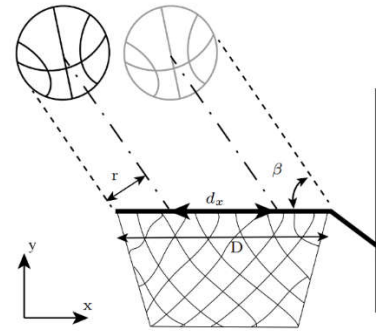


Figure 2. The success of throw based on the final coordinate and angle of approach towards hoop

This problem is formulated as:

$$\begin{aligned} \text{minimize } & \mu_p + 3\sigma_p \quad (3) \\ & 6 \leq V \leq 8 \\ & 60 \leq \theta \leq 70 \\ & W \sim \mathcal{N}(3,1) \\ & y \sim \mathcal{N}(2.3,0.1) \end{aligned}$$

In the following section, the steps for robust optimization using Robustimizer is explained.

### 1.1. Sensitivity analysis

The sensitivity analysis step is an optional step to find the magnitude of the effects of each parameter on each response. This step can help

<sup>1</sup> O. Nejadseyfi, H.J.M. Geijselaers, A.H. Boogaard, (2019). Robust optimization based on analytical evaluation of uncertainty propagation. Engineering Optimization, 51(9), 1581-1603.

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to eliminate the parameters that do not have significant influence on the output. The default number of design variables for sensitivity analysis is two. Enter the lower and upper bound values for each design variable using the data in Table 1. It is possible to rename the variables as shown in Figure 3.

Figure 3. Defining design variables for sensitivity analysis

As there is no correlation between the magnitude of wind and the throw height, choose the first option in the button group and keep the number of noise variables on two. Then enter the mean and standard deviation for each noise variable. To modify the names of the variables click on the checkbox and enter new names (Figure 4).

Figure 4. Defining noise variables for sensitivity analysis

Under *Create sensitivity DOE and blackbox evaluation* tab, click on *Create screening DOE* button. The DOE will be shown in the table. Save this DOE using *Save screening DOE* button. This tab-separated txt file includes the DOE points for sensitivity analysis. Use this DOE for evaluating the blackbox function. Evaluating the blackbox function is performed outside Robustimizer. In general it can be the results of finite element simulations, analytical models, results of solving differential equations, etc and it depends on the model made for representing the problem.

In this problem, the final coordinates at the height of the hoop and angle of approach towards hoop are calculated on DOE points by solving differential equation for projectile motion. Then the output is calculated using Equation (1) and is written to a txt file. If more

than one response is present, each column should provide the values for each response. Note that the number of rows should be equal to the number of DOE points.

	Velocity	Angle	WindSpeed	ThrowHt
DOE Sensitivity#1	6.0000	60.0000	0	2.0000
DOE Sensitivity#2	6.0000	60.0000	6.0000	2.6000
DOE Sensitivity#3	6.0000	70.0000	0	2.6000
DOE Sensitivity#4	6.0000	70.0000	6.0000	2.0000
DOE Sensitivity#5	8.0000	60.0000	0	2.6000
DOE Sensitivity#6	8.0000	60.0000	6.0000	2.0000
DOE Sensitivity#7	8.0000	70.0000	0	2.0000
DOE Sensitivity#8	8.0000	70.0000	6.0000	2.6000

Figure 5. Creating screening DOE

	Main Response
DOESens#1	367.1027
DOESens#2	6.9978
DOESens#3	28.8689
DOESens#4	111.9207
DOESens#5	121.2756
DOESens#6	97.6795
DOESens#7	0.5801
DOESens#8	5.6630

Figure 6. Importing blackbox evaluation results

Under *Sensitivity results* tab, select a response and click on *Run sensitivity analysis* button as shown in Figure 7. The percentage of influence of each input variable on the selected response will be shown in the results area (Figure 8). As all variables have a significant influence on the main

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response, all those variables will be included in robust optimization.

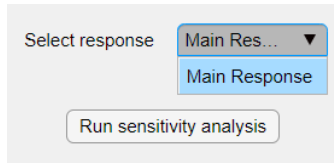


Figure 7. Performing sensitivity analysis

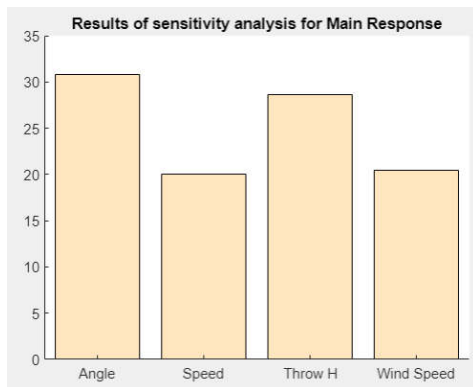


Figure 8. Results of sensitivity analysis

### 1.2. Defining input and output

The definition of robust optimization problem starts from this section. Change the number of design variables to two, the table will automatically resize to fit the new data. Enter the lower and upper bound values for each variable. It is possible to rename the variables as shown in Figure 9.



Figure 9. Entering number of design variables, their bound and names

As there is no correlation between the magnitude of wind and the throw height, choose the first option in the button group and modify the number of noise variables. Then enter the mean and standard deviation for each noise variable in the table. To modify the names of the variables click on the checkbox and enter new names.

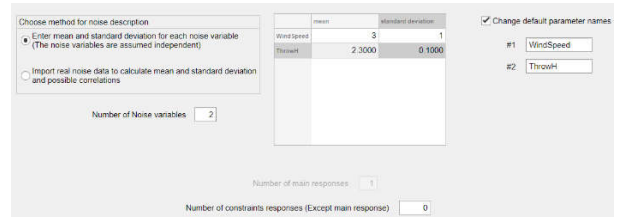


Figure 10. Entering number of noise variables, their statistics and names

### 1.3. Creating DOE and importing the blackbox function analysis

Enter the number of DOE points in the *DOE Size* field (Figure 11). The default method to make a DOE is LHS and the user can choose to combine the factorial design and LHS by clicking on the checkbox. After clicking on *Create DOE* button, the DOE is made and is displayed in the table as shown in Figure 12. Save the DOE as shown in Figure 13 as a txt file which includes the DOE points in a tab-separated format. Use this DOE for evaluating the blackbox function. Evaluating the blackbox function is performed outside Robustimizer. In general it can be the results of finite element simulations, analytical models, results of solving differential equations, etc and it depends on the model made for the problem.

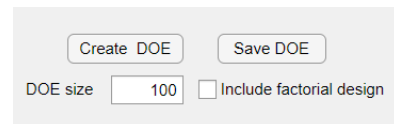


Figure 11. Creating DOE

	Velocity	Angle	WindSpeed	ThrowH
DOE#1	6.0000	60.0000	0	2.0000
DOE#2	6.0000	60.0000	6.0000	2.6000
DOE#3	6.0000	70.0000	0	2.6000
DOE#4	6.0000	70.0000	6.0000	2.0000
DOE#5	8.0000	60.0000	0	2.6000
DOE#6	8.0000	60.0000	6.0000	2.0000
DOE#7	8.0000	70.0000	0	2.0000
DOE#8	8.0000	70.0000	6.0000	2.6000
DOE#9	6.2432	64.0701	3.5954	2.4996
DOE#10	7.3550	68.1454	0.5583	2.3112
DOE#11	6.0063	67.4081	2.5783	2.3310
DOE#12	6.9564	69.1415	5.5262	2.5648
DOE#13	7.2377	69.4872	2.1610	2.0793
DOE#14	6.3473	64.8348	4.2327	2.0121
DOE#15	6.7804	66.8536	5.9229	2.0698
DOE#16	6.3714	69.9824	1.4173	2.5769
DOE#17	6.2333	63.7945	5.9461	2.1013
DOE#18	6.2796	65.5576	4.8227	2.2800
DOE#19	7.2945	60.9324	3.1266	2.3770

Figure 12. Displayed DOE in the table

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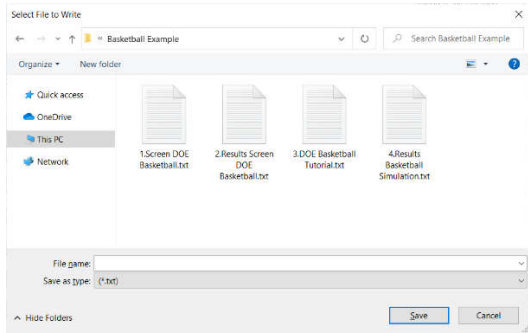


Figure 13. Saving DOE

File	Edit	Format	View	Help
6.000000	60.000000	0.000000	2.000000	
6.000000	60.000000	6.000000	2.600000	
6.000000	70.000000	0.000000	2.600000	
6.000000	70.000000	6.000000	2.000000	
8.000000	60.000000	0.000000	2.600000	
8.000000	60.000000	6.000000	2.000000	
8.000000	70.000000	0.000000	2.000000	
8.000000	70.000000	6.000000	2.600000	
6.243239	64.070115	3.595397	2.499566	
7.354997	68.145393	0.558310	2.311152	
6.006305	67.408133	2.578302	2.330987	
6.956388	69.141508	5.526225	2.564823	
7.237666	69.487176	2.160994	2.079350	
6.347348	64.834799	4.232672	2.012083	
6.780435	66.853623	5.922904	2.069842	
6.371414	69.862445	1.417345	2.576932	
6.233340	63.794545	5.946099	2.101308	
6.279583	65.557628	4.822677	2.280018	
7.294462	60.932386	3.129604	2.377024	
6.000000	60.000000	5.770000	2.400000	

Figure 14. Saved DOE

In this problem the final coordinate at the height of the hoop and angle of approach towards hoop on DOE points are calculated by solving differential equation for projectile motion considering initial velocity, angle of throw, throw height and air resistance due to wind. This step happens outside Robustimizer. It is possible to introduce a script to Robustimizer that automatically generates and imports the results. This topic is explained in another example.

The results of the black-box simulations are displayed in Figure 15. Then the output is calculated using Equation (1) and is written to a txt file as shown in Figure 16. If more than one response is present, each column should provide the values for each response. Note that the number of rows should be equal to the number of DOE points. The results of any simulation is obtained outside Robustimizer.

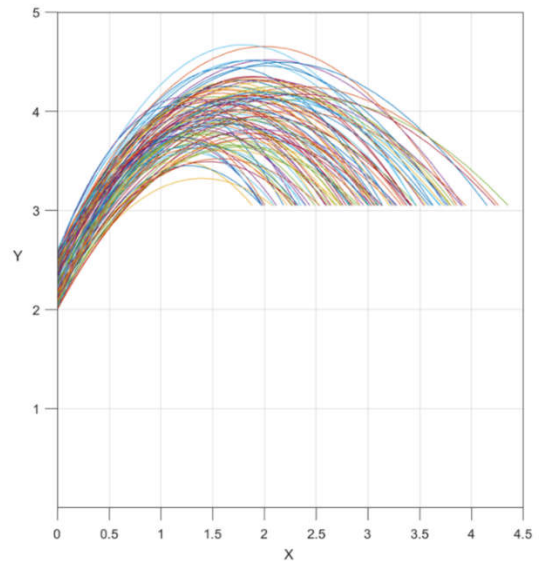


Figure 15. Results of blackbox function evaluation

```

File Edit Fgmat View Help
367.102698210244
6.99782787303512
28.8688888807000
111.920677330331
121.275581089716
97.6795341644203
0.580080242632830
5.66302096008247
6.71769701650916
0.0157885538599773
31.5421678351139
2.40754135831121
3.27844885967026
41.7344360926411
12.9263250400541
15.2624808101451
50.5858479640818
18.8994809281556
23.5266917068212
36.3410000000000
  
```

Figure 16. The output as a txt file

To import the results file to Robustimizer, select *Import from file* and then click on *Read results file* button. A new window appears to select the file (Figure 18). After choosing the file, the results will appear in the table as shown in Figure 19. As explained earlier, the user is capable of introducing a script file and Robustimizer can automatically run the script and import the results.

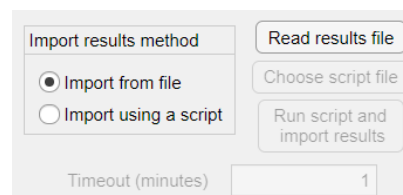


Figure 17. Options for importing the results

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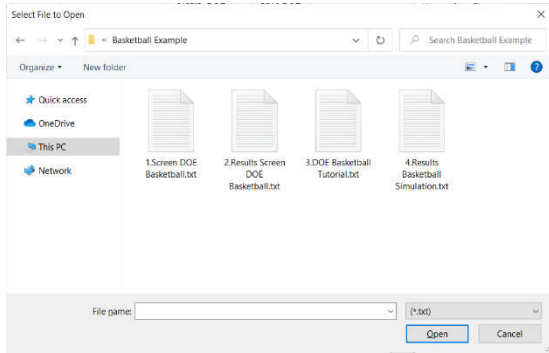


Figure 18. Importing results of blackbox evaluation

	Main Response
DOE#1	367.1027
DOE#2	6.9978
DOE#3	28.8689
DOE#4	111.9207
DOE#5	121.2756
DOE#6	97.6795
DOE#7	0.5801
DOE#8	5.6630
DOE#9	6.7177
DOE#10	0.0158
DOE#11	31.5422
DOE#12	2.4075
DOE#13	3.2784
DOE#14	41.7344
DOE#15	12.9263
DOE#16	15.2625
DOE#17	50.5858
DOE#18	18.8995
DOE#19	23.5267

Figure 19. The results of blackbox evaluation after importing

### 1.4. Fitting, validating and visualizing the metamodel

In this tab choose a metamodel type and click on *Fit metamodel* button. If fitting is successful a message will appear (Figure 20). To perform cross validation, use *Perform cross validation* button. For a large DOE this can be a time-consuming process and the progress can be followed using a wait bar shown in Figure 21. The user can view different results for cross validation for different responses using drop-down lists as shown in Figure 22.

To visualize the metamodel, use the *Plot metamodel* panel. It must be noted that the response is plotted versus two inputs at a time (3D plot). Therefore, if the total number of input parameters is more than two, the other parameters will be set on a nominal value. The nominal value for those variables that are out of X and Y axes, can be adjusted using the sliders

next to the plot (Figure 23). The user can select the inputs on the X and Y axes and the output on Z axis using drop-down lists. Subsequently, the sliders of those inputs on X and Y axis will be disabled and the other sliders will be enabled to assign the nominal values of other parameters. Press update plot to apply changes in sliders.

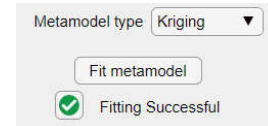


Figure 20. Fitting a metamodel

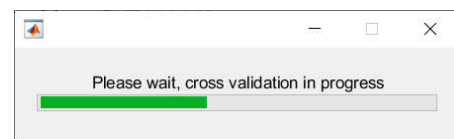


Figure 21. progress bar for validating a metamodel

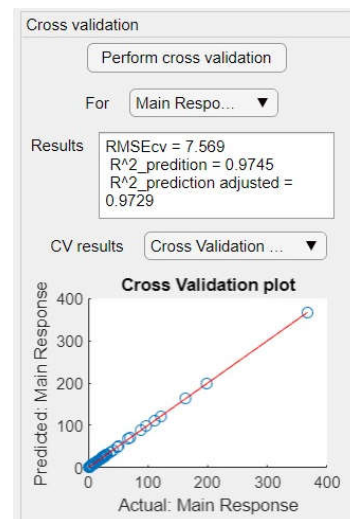


Figure 22. validating a metamodel

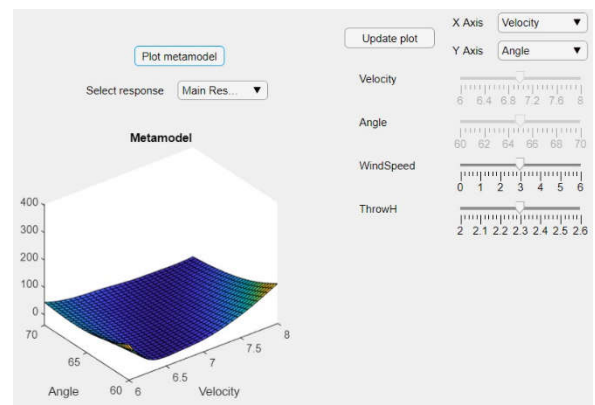


Figure 23. Visualization of metamodel



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### 1.5. Optimization settings

The user can proceed with the robust optimization without changing the default values in this tab. However, to have more control over the optimization procedure there are multiple options to choose from (Figure 24). Under *Optimization method* panel the user can choose SQP or Interior point method which are gradient-based methods of optimization. In addition, Genetic algorithm (GA) method can be used in which gradients are not evaluated. If GA is selected, the populations size and generation size must be chosen to perform optimization. Usually SQP is the fastest method to find the optimum.

Noise propagation can be performed using analytical method or Monte Carlo analysis. For Monte Carlo, a sample is required to perform noise propagation during robust optimization. This sample can be generated using random sampling from a Gaussian distribution. As this sample is random and each time the *Create MC sample* button is pushed a new sample is generated, the user can save and load the MC sample to obtain a consistent result. It is also possible to create MC sample using a different criterion (e.g. Real Data, non-normal data, skewed distribution, etc) outside Robustimizer and import a tailor-made MC Sample as a txt file (Figure 25).

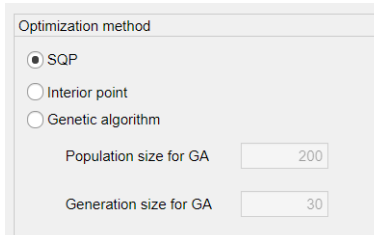


Figure 24. Optimization method panel

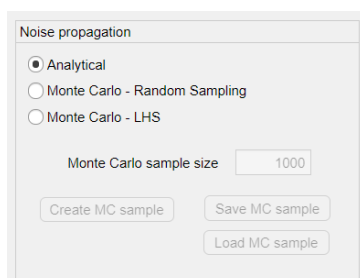


Figure 25. Noise propagation panel

Objective function for the main response can be modified in *Objective function* panel. Several criteria exist to choose from. A target value is required for specific objectives. The user can choose to enhance the search for robust optimum design by accounting for the skewness of the response by clicking in the tick box (Figure 26).

If constraints are present in the problem, the settings for each constraint can be changed in *Constraints* panel. As there are no constraints in this problem, this panel is not editable.

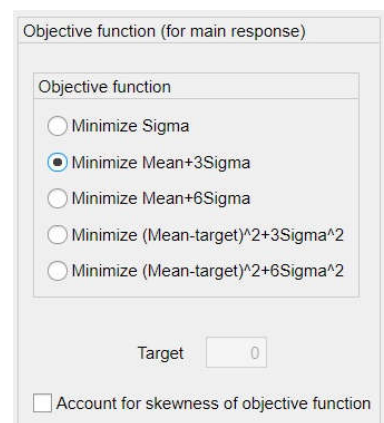


Figure 26. Objective function definition panel

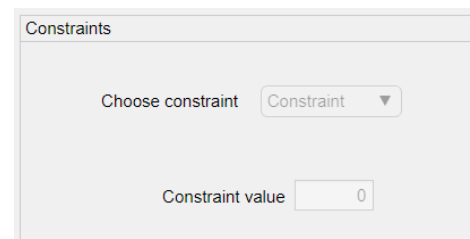


Figure 27. Constraints definition panel

### 1.6. Robust optimization

Click on *Perform optimization* button to start the optimization procedure. The required time for optimization varies depending on the size of the problem, size of the DOE and the options chosen under the *Optimization settings* tab. After optimization is done and an optimum is found, the results are displayed in the textbox as shown in Figure 28. To view the scatter around the robust optimum design, click on *Perform Monte Carlo analysis on optimum* button shown in Figure 29.

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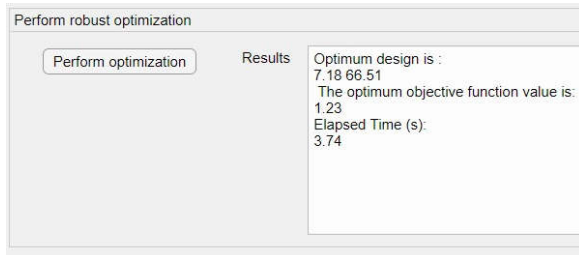


Figure 28. Results of robust optimization

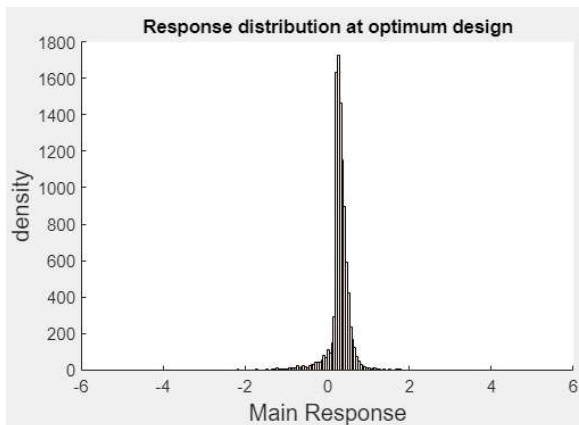


Figure 29. Scatter around the optimum

### 1.7. Sequential improvement of the metamodel

Choose a method to add an infill point to the current DOE. As mentioned earlier, adding new DOE point can be done either manually or automatically. Since no script is introduced under the *DOE and blackbox evaluation* tab, only manual option is applicable. For this purpose, click on *Recommended an infill point for manual blackbox evaluation* button. A wait bar will show the progress of adding the infill point as shown in Figure 30. The recommended infill point will be written in the text area Figure 31. Then select, copy and paste the new infill point at the end of the DOE txt file and import the updated DOE in the *DOE and blackbox evaluation* tab. Alternatively, click on *Save old output and add new infill point to existing DOE* button. This will automatically add the infill point to the DOE table (Figure 32) and save the existing output as a txt file in the selected folder.

It is then required to perform the blackbox evaluation on the new infill point, add the results to the end of the results txt file and import it in the *DOE and blackbox evaluation* tab. The size of new DOE and new results file increases by one row.

After updating the DOE and importing the results, repeat the process of robust optimization from fitting a metamodel, and perform the robust optimization again.



Figure 30. progress of search for the best infill point

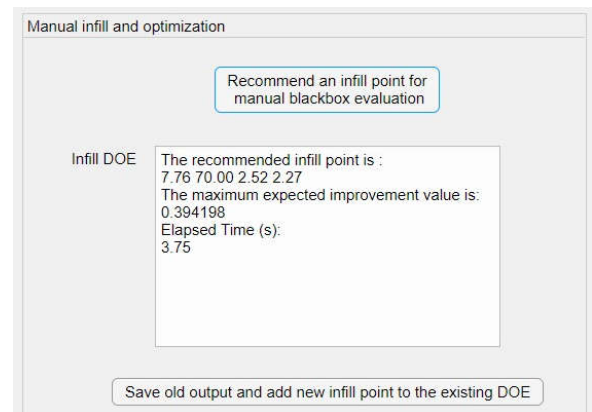


Figure 31. The recommended infill point

	Velocity	Angle	Wind Speed	Turnout	Main Response
DOE001	61.2170	63.0434	4.2000	2.3831	0
DOE002	6.0784	62.3638	1.1880	2.9159	0
DOE003	6.2690	60.9681	0.7206	2.4428	0
DOE004	7.4890	65.5283	1.4703	2.0977	0
DOE005	7.2805	67.5256	0.1101	2.0877	0
DOE006	7.4414	66.1159	4.4592	2.0148	0
DOE007	7.8405	63.5815	3.0334	2.3623	0
DOE008	6.9639	66.4696	3.1416	2.3958	0
DOE009	7.1708	63.6242	2.2450	2.1405	0
DOE010	6.6385	67.8829	2.3386	2.0382	0
DOE011	6.8651	68.0169	4.8242	2.1615	0
DOE012	7.5130	64.7702	0.4650	2.3224	0
DOE013	7.6995	61.6917	2.0130	2.1082	0
DOE014	7.5823	62.7902	4.8955	2.4546	0
DOE015	7.9141	69.3036	1.3187	2.5212	0
DOE016	7.3318	60.8395	1.9989	2.2307	0
DOE017	7.6845	65.8765	4.1344	2.1537	0
DOE018	6.6216	62.7004	0.8241	2.3487	0
DOE019	7.7615	70.0000	2.6168	2.2668	0

Figure 32. New infill point added at the end of the existing DOE

To validate the obtained robust optimum, the blackbox function can be evaluated on the obtained design setting multiple times to check the validity of the results.